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13. ABSTRACT (Maximum 200 words) Our long-term objective has been the development of a high-fidelity and high-performance simulation capability for predicting and optimizing the dynamic aeroelastic response of a fighter during three-dimensional high-C maneuvers in subsonic, transonic, and supersonic airstreams. Our focus has been on Air Force problems involving a modern fighter or bomber, and relevant to new approaches for flutter testing, mitigation of limit-cycle (LCO) and pilot induced (PIO) oscillations, as well as performance optimization. Our starting point has been the unique aeroelastic simulation capability developed at the University of Colorado under the sponsorship of the Air Force office of Scientific Research, and in partnership with the Flight Test Center at the Edwards Air Force Base.				
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**AIR FORCE OFFICE OF SCIENTIFIC RESEARCH  
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Final Report - October 2001**

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**SUMMARY**

This is a three-part final report on the research supported by the Air Force Office of Scientific Research between 1998 and 2001, under Grant F49620-99-1-0007 entitled **Simulation of the Transient Aeroelastic Response of a Realistic Aircraft Configuration During Three-Dimensional High G Maneuvers**.

**1. Motivations and research plan**

Our long-term objective has been the development of a high-fidelity and high-performance simulation capability for *predicting* and *optimizing* the dynamic aeroelastic response of a fighter during three-dimensional high-G maneuvers in subsonic, transonic, and supersonic airstreams. Our focus has been on Air Force problems involving a modern fighter or bomber, and relevant to new approaches for flutter testing, mitigation of limit-cycle (LCO) and pilot induced (PIO) oscillations, as well as performance optimization. Our starting point has been the unique aeroelastic simulation capability developed at the University of Colorado under the sponsorship of the Air Force Office of Scientific Research, and in partnership with the Flight Test Center at the Edwards Air Force Base.

In order to achieve the long term-objective outlined above, we had defined the following milestones that have constituted our short-term objectives during the three years of funding.

- Develop a high-performance finite element corotational methodology aimed at capturing correctly the nonlinear geometric effects during the simulation of complex aircraft

maneuvers. Indeed, during a realistic three-dimensional high-G maneuver, an aircraft undergoes *large displacements and rotations*. Therefore, in that case, simulating the dynamic behavior of the flexible airframe cannot be limited to a vibrational structural analysis around a fixed equilibrium point, as usually done for flutter analysis. Rather, the entire motion of the structure must be tracked, which requires introducing geometric stiffnesses in the structural model and performing a nonlinear analysis. Such a nonlinear analysis increases the computational complexity of an aeroelastic simulation, and therefore calls for a fast and scalable nonlinear structural solver.

- Develop a versatile methodology for designing the control laws needed for implementing a specific maneuver, and simulating the corresponding actuation of the control surfaces of an aircraft. Before this research, all maneuvers have been simulated either in a quasi-static fashion, or by driving some selected points of the aircraft. The former approach is inaccurate in general, and fails to capture transient effects in particular. The latter approach is not reliable for the prediction of the stresses and strains that develop during a high-G maneuver in the airframe. Piloting the maneuver through intelligent inputs to the control surfaces is the only reliable method for simulating the aeroelastic behavior of an aircraft during flight. However, such an approach, which needed to be developed and investigated, requires the re-engineering of existing flow solvers and dynamic mesh algorithms to address the mathematical and practical consequences on the flow variables and fluid grid behavior during the opening and closing of the control surfaces.
- Develop a fast, robust, and general purpose mesh motion scheme for flow problems with moving and deforming boundaries. Previous work in this area has been mostly based on particular instances of the concept of a virtual elastodynamic fluid grid, and targeted and applied to small-amplitude mesh motions. During the previous funding period, the PI and his co-workers had improved earlier the lineal spring analogy method by introducing the concept of torsional springs, which unlike any other proposed mesh motion algorithm, is mathematically guaranteed to prevent crossovers during an arbitrary mesh motion. However, even such a robust algorithm is not suitable for maneuvers because in that case, the structure undergoes *large displacements and large rotations*. For such applications, it is essential to extract first the rigid body component of the motion of the flexible aircraft, apply it to predict the global position of the fluid dynamic mesh, and then correct the instantaneous position of the fluid grid by deforming it according to the deformational component of the motion of the flexible aircraft. Other mesh motion related problems that needed to be addressed include the handling of mesh shearing during the opening and closing of control surfaces, and the automatic treatment of far-field boundary conditions. All these issues must be investigated and resolved before high-G three-dimensional maneuvers can be accurately simulated.
- Adapt and tailor various turbulence models and wall laws for aeroelastic applications. More specifically, expand the mathematical formulations of these models and laws as

well as their corresponding discretization schemes to the case of moving grids, tune their various parameters to the applications of interest, and design efficient implementations on high-performance parallel computers.

- Develop an intrinsically parallel fluid/structure partition analysis procedure for speeding up the simulation of the aeroelastic response of fighters during complex maneuvers. Indeed, before this research, our fluid/structure coupled solution algorithm — as well as most if not all published algorithms — has been inherently sequential. While it has allowed for *intra-parallelism* — that is, for parallel computations within the fluid and structure analyzers — it did not allow for *inter-parallelism* between the fluid and structure computations. The fluid system had to be updated before the structural system could be advanced. For level flight applications where the structure remains in the linear regime, this is not a serious limitation because the computational cost associated with the structural analyzer is negligible compared to that of the flow analyzer. However, as stated earlier, maneuvering applications call for nonlinear structural analyses that increase significantly the computational cost of an aeroelastic simulation. Therefore, our objective has been to develop a computationally efficient, and yet higher-order time-accurate staggered procedure for solving the coupled fluid/structure equations of motion that features *inter-parallelism* in addition to *intra-parallelism*, and which therefore allows advancing simultaneously in time the structure and flow state variables without a significant loss of accuracy.
- Incorporate the treatment of thermal effects in our aeroelastic solution methodology. Indeed, the temperature of the skin of an aircraft,  $T_{skin}$ , grows with the free-stream Mach number  $M_\infty$  as follows

$$T_{skin} = T_\infty \left( 1 + \sqrt{P_r} \frac{\gamma - 1}{2} M_\infty^2 \right)$$

where  $T_\infty$  is the free-stream temperature,  $P_r$  is Prandtl's number, and  $\gamma$  is the specific heat ratio. Hence, at the higher supersonic Mach numbers,  $T_{skin}$  increases, and the thermal effects become important for the prediction of the behavior of both the fluid and the structure.

- Augment our aeroelastic simulation technology with a high-performance and high-accuracy computational framework for performing the sensitivity analysis of a given coupled aeroelastic system. Such a framework is to feature analytical derivations of the gradients of the relevant semi-discrete operators, rather than lower-order finite difference versions, address both direct and adjoint solution methods, and include efficient staggered procedures for solving all intermediate coupled systems of equations. Its main objective is to expand the range of applications of our aeroelastic simulation capability to the aeroelastic — rather than aerodynamic or structural — optimization of fighters based on nonlinear flow and structural mechanics theories.

## **2. Achievements**

During the three-year funding period, we have made significant accomplishments in the following areas pertaining to our short- and long-term objectives described above.

### **2.1. A corotational finite element method for aircraft maneuvering applications**

We have developed a systematic approach to the element-independent corotational dynamics of finite elements that eases the implementation and execution of the geometrically nonlinear analysis of aircraft structures. We have successfully applied our formulation to the simulation of various pull and roll maneuvers of complete fighter configurations where the spars, ribs, skin, fuselage, hinges, control surfaces, and discrete masses are modeled by suitable finite elements. This achievement is documented in our publications [6,19,23,27].

### **2.2. A two-scale mesh motion algorithm for simulating aircraft maneuvers**

Torsional springs provide great robustness for flutter and/or aeroelastic computations where the fluid mesh undergoes large deformations. However, these springs and most if not all other elasticity based mesh motion schemes are neither sufficiently reliable nor sufficiently performant when the structure undergoes large displacements and rotations, as in maneuvering. For this reason, we have also designed a new corotational-like mesh motion strategy where the motion of the surface of the structure is first decomposed into a rigid body component and a deformational one. First, the rigid body component is transferred to the fluid mesh using simple translations and rotations. Then, the deformational component is applied as a boundary condition to the fluid mesh system, which is then relaxed to achieve equilibrium using the torsional springs. This strategy has enabled the simulation of high-angle pitching and rolling of complete flexible fighter configurations, and has speeded-up simpler simulations that were possible using classical techniques by a factor ranging between 2 and 10, depending on some key configuration parameters. It has been summarized in our publications [6,19,23,25].

### **2.3. Further mathematical development of the Discrete Geometric Conservation Laws**

We have established a firm theoretical basis for the enforcement of Discrete Geometric Conservation Laws (D-GCLs) while solving flow problems with moving meshes. The GCL condition governs the geometric parameters of a given numerical solution method, and requires that these be computed so that the numerical procedure reproduces exactly a constant solution. Previously, we have shown that this requirement corresponds to a time-accuracy condition. More specifically, we have proved that satisfying an appropriate D-GCL is a sufficient condition for a numerical scheme to be at least first-order time-accurate on moving meshes. While this results sheds some light on the theoretical status of the D-GCL, it does not fully explain why it has been reported in the literature that violating

the D-GCL introduces a weak instability in the numerical solution on moving grids of Euler flows. We have also proved that satisfying an appropriate D-GCL is a necessary and sufficient condition for a scheme to be unconditionally stable on moving grids. This new result sheds a new light on the relationship between the D-GCL and numerical scalability, and provides new means for predicting the behavior of a numerical scheme when it does not satisfy its corresponding D-GCL. We have documented this achievement in our publications [2,10,14,18,24].

#### **2.4. A third-order energy-accurate partitioned procedure for the fully parallel solution of coupled fluid-structure interaction problems**

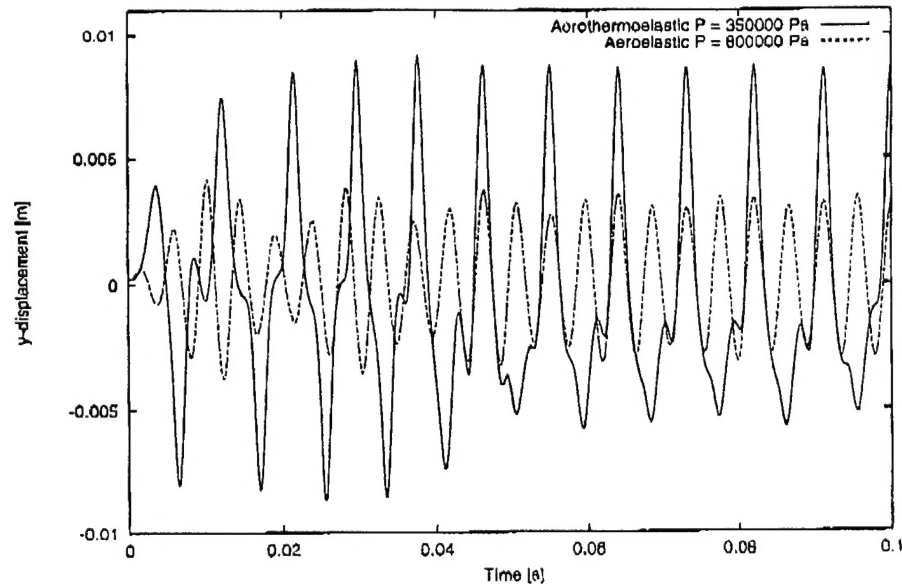
We have developed a mathematical framework for assessing some important numerical properties of the partitioned procedure chosen for solving a coupled fluid/structure system of equations, and predicting its performance for realistic applications. Our analysis framework is based on the estimation of the energy that is artificially introduced at the fluid/structure interface by the staggering process that is inherent to most partitioned solution methods. This framework also suggests alternative approaches for time-discretizing the transfer of aerodynamic data from the fluid subsystem to the structure subsystem that improves the accuracy and stability properties of the underlying partitioned method. We have applied this framework to the analysis of several partitioned procedures that have been previously proposed for the solution of nonlinear transient aeroelastic problems. Using two- and three-dimensional, transonic and supersonic, wing and panel aeroelastic applications, we have validated this framework and highlight its impact on the design and selection of a staggering algorithm for the solution of coupled fluid/structure equations. Most importantly, we have also exploited the mathematical features of this framework to design a new, third-order energy-accurate, partitioned procedure for the solution of nonlinear transient aeroelastic problems that features both inter- and intra-parallelism. For maneuvering applications where the structure requires a geometrically nonlinear analysis, this new algorithm which allows advancing simultaneously in time the structure and flow state variables has improved computational speed by a factor two, as expected. This achievement is documented in our publications [8,11,12,16,21].

#### **2.5. A four-field formulation for the solution of aerothermoelastic problems**

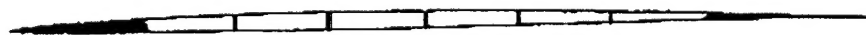
The main computational difficulty in incorporating thermal effects in our nonlinear aeroelastic solution methodology has been the accurate and efficient solution of the four coupled partial differential equations that govern the fluid, the structure, the heat transfer in the structure, and the mesh motion. To address this challenging problem, we have extended our three-way coupled formulation of fluid/structure interaction problems to a four-field formulation: the fluid, the structure, the fluid dynamic mesh, and heat transfer. In this new approach, the wall boundary of the flow is no longer assumed to be isothermal or adiabatic. Rather, the temperature distribution on the surface of the structure is considered as an unknown that is determined by enforcing the continuity of the temperature



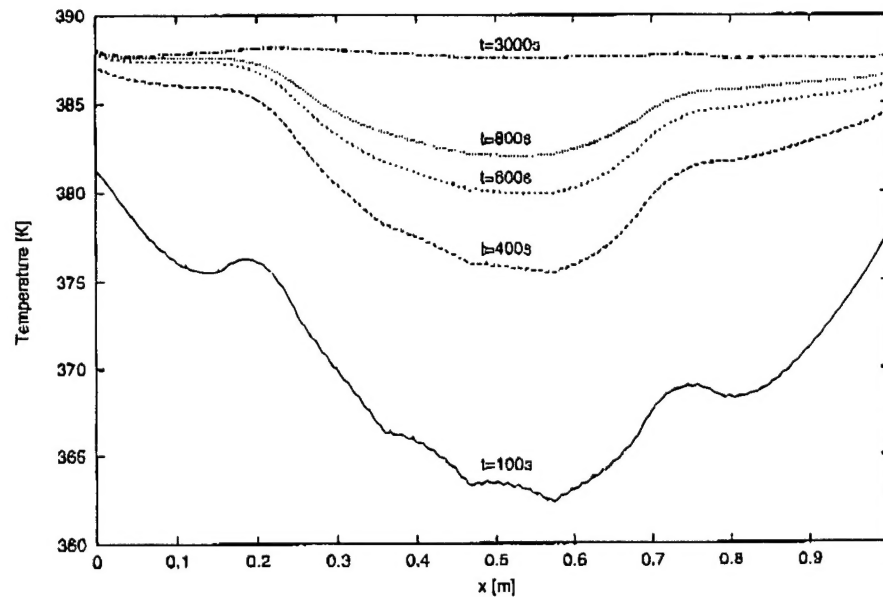
field and temperature fluxes at the fluid/structure interface boundary. For two-dimensional problems, we have developed the semi-discrete equations governing this four-field problem, and constructed several partitioned procedures for time-integrating them. We have applied this four-field formulation to the aerothermoelastic stability analysis of flat panels in supersonic turbulent flows, and to the thermal analysis of an F-16 multicell wing section partially filled with kerosene. We have documented this achievement in our publication [26].



Influence of thermal effects on aeroelastic loads  
(F-16 typical wing section)



F-16 multicell wing section



Time evolution of the temperature distribution in the F-16 skin  
(taking into account the presence of fuel)

## 2.6. Analytically based sensitivity analysis and optimization of nonlinear aeroelastic systems

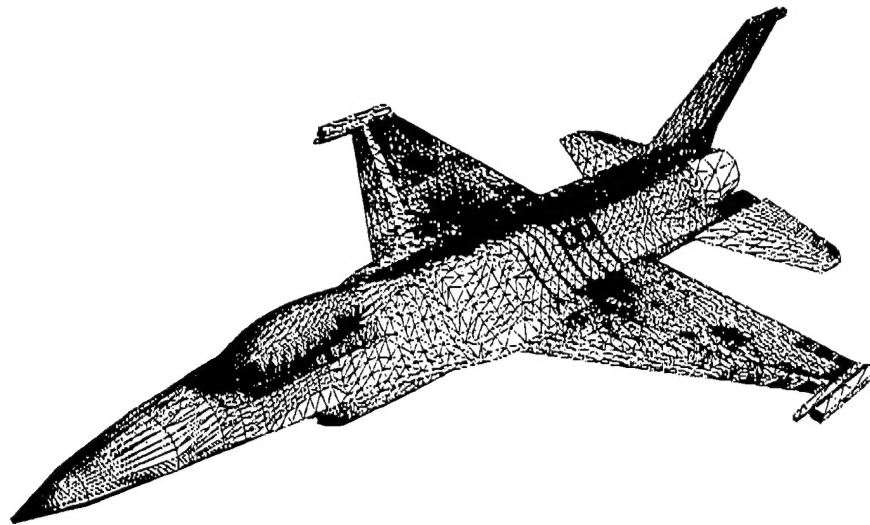
We have considered the problem of optimizing for steady-state conditions a given aeroelastic system by varying both aerodynamic and structural parameters such as the shape of the dry or wet surface, and the orientation of the composite fibers. We have developed a solution methodology for this optimization problem that is based on a sequential quadratic programming procedure where the gradients of the optimization criteria with respect to the optimization variables are determined by an analytical approach. In this methodology which features both the direct and adjoint approaches for optimization, we evaluate at each optimization step the aeroelastic steady-state response of the system using a staggered procedure that couples efficiently a finite element solution method for the structure subsystem and a 3D Euler finite volume method for the fluid subsystem. We have illustrated this optimization methodology with several three-dimensional examples and documented it in our publications [4,5,17,20].

## 2.7. Simulation of the aeroelastic behavior of complete F-16 configurations and validation

We have applied our simulation technology to the flutter clearance of an F-16 Block 40 in clean wing configuration but with tip missiles, for  $0.7 \leq M_\infty \leq 1.4$  at the altitude of 3,000 m. Based on modeling information provided by Lockheed-Martin, we have constructed at the University of Colorado a detailed but undamped three-dimensional FE structural dynamics model for the F-16 Block 40 in clean wing configuration but with a



missile and launching system at each wing tip. This FE model features bar, beam, solid, plate, shell, metallic as well as composite elements, and a total of 168,799 dofs. It reproduces correctly the first ground bending and torsion frequencies which were measured as 4.76 Hz and 7.43 Hz, respectively. Using F-16 CAD data provided by the Air Force Research Laboratory at Wright Patterson and ignoring the wing tip missiles, we have generated a surface grid with 63,044 grid points then an Euler fluid volume mesh with 403,919 vertices. To determine the bending and torsional aeroelastic parameters, we have employed a procedure replicating what occurs in flight testing. We have excited the structure in an appropriate manner and simulated its response to the prescribed initial disturbance. For each different Mach number, this generated 168,799 signals, one for each dof of the detailed FE structural model. We have computed these signals for a little more than two cycles, then applied the Eigensystem Realization Algorithm to extract from a few of them --- representing sensor information --- the sought-after aeroelastic parameters. We have compared our results to flight test data provided by the Edward Air Force Base and observed less than 7% relative errors. This very good correlation has attracted the attention of major aerospace companies with whom we are now working on further validations.

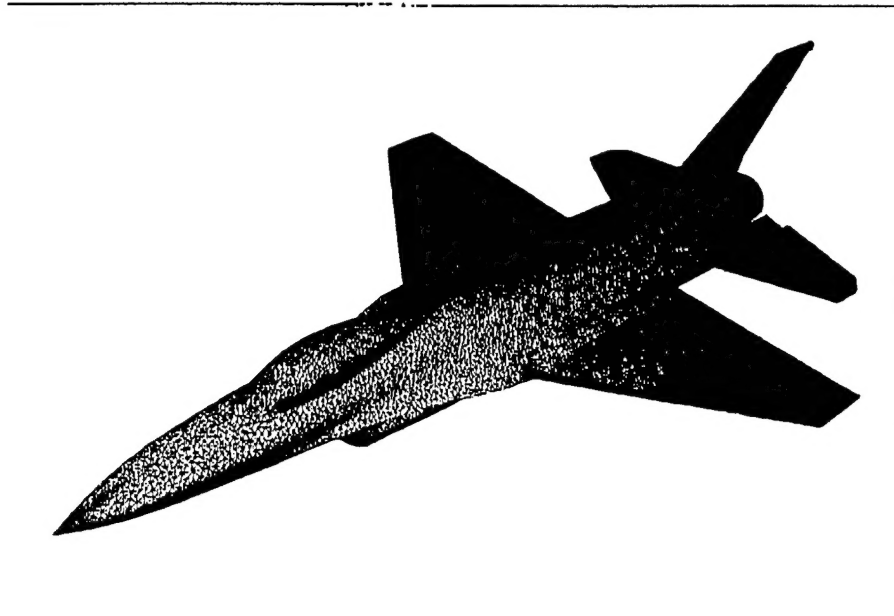


Detailed finite element structural model of an F-16 Block 40

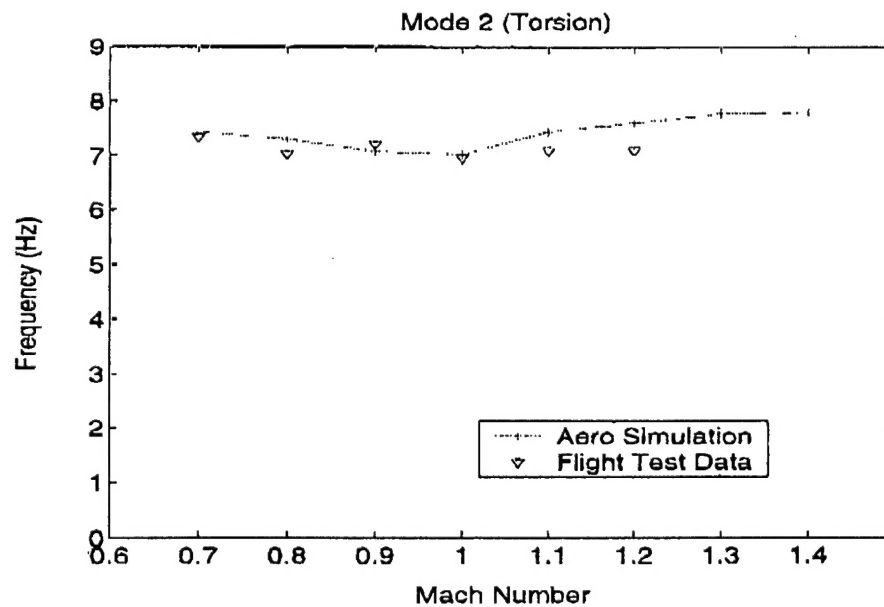
### 3. Publications that have resulted from the support by this Grant

#### Monographs and Book Chapters

1. C. Farhat, B. Koobus, and H. Tran, "Simulation of Vortex Shedding Dominated Flows Past Rigid and Flexible Structures," *Computational Methods for Fluid-Structure Interaction*, ed. T. Kvamsdal, I. Enevoldsen, K. Herfjord, C. B. Jenssen, K. Mehr and S. Norsett, Tapir, pp. 1-30 (1999)



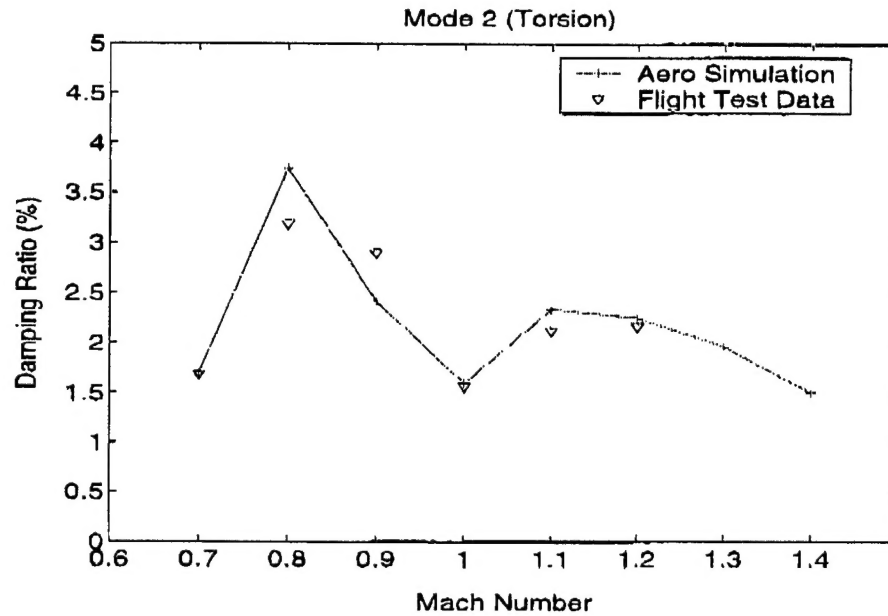
CFD surface grid for an F-16 Block 40



Aeroelastic torsional frequency for an F-16 Block 40 at an altitude of 3,000 m

### Refereed Journals

2. C. Farhat, P. Geuzaine and C. Grandmont, "The Discrete Geometric Conservation Law and the Nonlinear Stability of ALE Schemes for the Solution of Flow Problems on Moving Grids," *Journal of Computational Physics*, (in press)
3. C. Farhat, P. Geuzaine and G. Brown, "Application of a Three-Field Nonlinear Fluid-Structure Formulation to the Prediction of the Aeroelastic Parameters of an F-16



Aeroelastic torsional damping for an F-16 Block 40 at an altitude of 3,000 m

Fighter," *Computers and Fluids*, (in press)

4. K. Maute, M. Nikbay and C. Farhat, "Coupled Analytical Sensitivity Analysis and Optimization of Three-Dimensional Nonlinear Aeroelastic Systems," *AIAA Journal*, (in press)
5. M. Lesoinne and C. Farhat, "A CFD Based Method for Solving Aeroelastic Eigenproblems in the Subsonic, Transonic, and Supersonic Regimes," *AIAA Journal of Aircraft*, Vol. 38, pp. 628-635 (2001)
6. C. Farhat, K. Pierson and C. Degand, "Multidisciplinary Simulation of the Maneuvering of an Aircraft," *Engineering with Computers*, Vol. 17, pp. 16-27 (2001)
7. C. Felippa, K. C. Park and C. Farhat, "Partitioned Analysis of Coupled Mechanical Systems," *Computer Methods in Applied Mechanics and Engineering*, Vol. 190, pp. 3247-3270 (2001)
8. S. Piperno and C. Farhat, "Partitioned Procedures for the Transient Solution of Coupled Aeroelastic Problems - Part II: Energy Transfer Analysis and Three-Dimensional Applications," *Computer Methods in Applied Mechanics and Engineering*, Vol. 190, pp. 3147-3170 (2001)
9. B. Koobus, H. Tran and C. Farhat, "Computation of Unsteady Viscous Flows Around Moving Bodies Using the  $k-\epsilon$  Turbulence Model on Unstructured Dynamic Grids," *Computer Methods in Applied Mechanics and Engineering*, Vol. 190, pp. 1441-1466 (2000)
10. H. Guillard and C. Farhat, "On the Significance of the Geometric Conservation Law for Flow Computations on Moving Meshes," *Computer Methods in Applied Mechanics and Engineering*, Vol. 190, pp. 1467-1482 (2000)

11. S. Piperno and C. Farhat, "Design of Efficient Partitioned Procedures for the Transient Solution of Aeroelastic Problems," *La Revue Européenne des Eléments Finis*, Vol. 9, No. 6/7, pp. 655-680 (2000)
12. C. Farhat and M. Lesoinne, "Two Efficient Staggered Procedures for the Serial and Parallel Solution of Three-Dimensional Nonlinear Transient Aeroelastic Problems," *Computer Methods in Applied Mechanics and Engineering*, Vol. 182, pp. 499-516 (2000)
13. B. Koobus and C. Farhat, "On the Implicit Time-Integration of Semidiscrete Viscous Fluxes on Unstructured Dynamic Meshes," *International Journal for Numerical Methods in Fluids*, Vol. 29, No. 8, pp. 975-996 (1999)
14. B. Koobus and C. Farhat, "Second-Order Time-Accurate and Geometrically Conservative Implicit Schemes for Flow Computations on Unstructured Dynamic Meshes," *Computer Methods in Applied Mechanics and Engineering*, Vol. 170, pp. 103-130 (1999)

### Refereed Proceedings

15. K. Maute, M. Nikbay and C. Farhat, "High-Performance Computing for the Optimization of Aeroelastic Systems, Proceedings of the First MIT Conference on Computational Fluid and Solid Mechanics, MIT, Cambridge, June 11-15 (2001)
16. S. Piperno and C. Farhat, "Design of Efficient Partitioned Procedures for Transient Nonlinear Aeroelastic Problems Based on Energy Exchange Criteria," Proceedings of the European Conference on Computational Mechanics (ECCM) 2001, Cracow, Poland, June 26-29 (2001)
17. K. Maute, M. Nikbay and C. Farhat, "Large-Scale Optimization of Aeroelastic Systems," in: Proceedings of the International Conference on Trends in Computational Mechanics, W. A. Wall, K. U. Bletzinger and K. Schweizerhof, eds., CIMNE, pp. 613-622 (2001)
18. C. Farhat, P. Geuzaine and C. Grandmont, "The Discrete Geometric Conservation Law and its Effects on Nonlinear Stability and Accuracy," *AIAA Paper 2001-2607, 15th AIAA Computational Fluid Dynamics Conference*, Anaheim, California, June 11-14 (2001)
19. C. Farhat, K. Pierson and C. Degand, "A CFD Based Simulation of the Unsteady Aeroelastic Response of a Maneuvering Vehicle," Proceedings of the European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS) 2000, Barcelona, Spain, September 11-14 (2000)
20. K. Maute, M. Nikbay and C. Farhat, "Analytically Based Sensitivity Analysis and Optimization of Nonlinear Aeroelastic Systems," *AIAA Paper 2000-4825, 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Long Beach, CA, September 6-8 (2000)
21. S. Piperno and C. Farhat, "Energy Based Design and Analysis of Staggered Solvers for Nonlinear Transient Aeroelastic Problems," *AIAA Paper 2000-1447, 41st AIAA/ASME/ASCE*

*Structures, Structural Dynamics, and Materials Conference*, Atlanta, GA, April 3-6 (2000)

22. K. Maute, M. Lesoinne and C. Farhat, "Optimization of Aeroelastic Systems using Coupled Analytical Sensitivities," *AIAA Paper 2000-0560, 38th Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, January 10-13 (2000)
23. C. Farhat, K. Pierson and C. Degand, "CFD Based Simulation of the Unsteady Aeroelastic Response of a Maneuvering Vehicle," *AIAA Paper 2000-0899, 38th Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, January 10-13 (2000)
24. H. Guillard and C. Farhat, "On the Significance of the GCL for Flow Computations on Moving Meshes," *AIAA Paper 99-0793, 37th Aerospace Sciences Meeting and Exhibit*, Reno, Nevada, January 11-14 (1999)

#### **Ph. D. Theses**

25. Christoph Degand, Ph. D. Thesis: Moving Grids for Nonlinear Dynamic Aeroelastic Simulations, Department of Aerospace Engineering Sciences, University of Colorado at Boulder (2001).
26. Hai Tran, Ph. D. Thesis: Numerical Simulation of Fluid/Structure Interaction Phenomena in Viscous Dominated Flows, Department of Aerospace Engineering Sciences, University of Colorado at Boulder (2001)
27. Kendall Pierson, Ph. D. Thesis: A Family of Domain Decomposition Methods for the Massively Parallel Solution of Computational Mechanics Problems, Department of Aerospace Engineering Sciences, University of Colorado at Boulder (2000).